DOCUMENT RESUME

ED 323 253 TM 015 492

AUTHOR Samejima, Fumiko
TITLE Differential Weight Procedure of the Conditional P.D.F. Approach for Estimating the Operating Characteristics of Discrete Item Responses.
INSTITUTION Tennessee Univ., Knoxville. Dept. of Psychology.
SPONS AGENCY Office of Naval Research, Arlington, VA. Cognitive and Neural Sciences Div.
REPORT NO ONR-RR-90-4; R01-1069-11-001-91
PUB DATE 25 Jun 90
CONTRACT N00014-87-K-0320-4421-549
NOTE 40p.
PUB TYPE Reports - Evaluative/Feasibility (142)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS Ability Identification; Adaptive Testing; Computer Assisted Testing; Equations (Mathematics); *Estimation (Mathematics); *Item Response Theory; *Mathematical Models; Simulation; Test Items
IDENTIFIERS *Differential Weight Procedure; *Operating Characteristics Estimation

ABSTRACT

A method is proposed that increases the accuracies of estimation of the operating characteristics of discrete item responses, especially when the true operating characteristic is represented by a steep curve, and also at the lower and upper ends of the ability distribution where the estimation tends to be inaccurate because of the smaller number of subjects in the base data. This technique is called the Differential Weight Procedure and belongs to the conditions P.D.F. approach. The method is illustrated with simulated data based on conventional testing, with 100 dichotomous items in 4 subtests of 25 items each given to 6,000 hypothetical examinees and adaptive testing in which each of 50 binary items has been given to 1,500 hypothetical subjects out of 15,000. Fifteen graphs illustrate these applications. The sensitivities of the resulting estimated operating characteristics to irregularities of the differential weight functions are discussed and usefulness of the method is reviewed. (SLD)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
☒ This document has been reproduced as received from the person or organization originating it
☐ Minor changes have been made to improve reproduction quality
• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy

ONR/RR-90-4

ED328258

DIFFERENTIAL WEIGHT PROCEDURE OF THE CONDITIONAL P.D.F. APPROACH FOR ESTIMATING THE OPERATING CHARACTERISTICS OF DISCRETE ITEM RESPONSES

FUMIKO SAMEJIMA

UNIVERSITY OF TENNESSEE

KNOXVILLE, TENN. 37996-0900

JUNE, 1990

Prepared under the contract number N00014-87-K-0398,
4421-549 with the
Cognitive Science Research Program
Cognitive and Neural Sciences Division
Office of Naval Research

Approved for public release; distribution unlimited.
Reproduction in whole or in part is permitted for
any purpose of the United States Government.

RR-1990-11-001-91

TM015492

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved OMB No 0704-0188

1a REPORT SECURITY CLASSIFICATION: Unclassified	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION / AVAILABILITY OF REPORT: Approved for public release; Distribution unlimited
2b DECLASSIFICATION / DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S)	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION: Fumiko Samejima, Ph.D. Psychology Department; 6b OFFICE SYMBOL (If applicable)	7a NAME OF MONITORING ORGANIZATION: Cognitive Science 1142 CS
6c ADDRESS (City, State, and ZIP Code): 310B Austin Peay Building The University of Tennessee Knoxville, TN 37996-0900	7b ADDRESS (City, State, and ZIP Code): Office of Naval Research 800 N. Quincy Street Arlington, VA 22217
8a NAME OF FUNDING / SPONSORING ORGANIZATION: Cognitive Science Research Program; 8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: N00014-87-K-0320
8c ADDRESS (City, State, and ZIP Code): Office of Naval Research 800 N. Quincy Street Arlington, VA 22217	10 SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO	PROJECT NO	TASK NO	WORK UNIT ACCESSION NO
61153N	RR-042-04	042-04-01	4421-549

11 TITLE (Include Security Classification): Differential Weight Procedure of the Conditional P.D.F. Approach for estimating the operating characteristics of discrete item responses

12 PERSONAL AUTHOR(S): Fumiko Samejima, Ph.D.

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
technical report	FROM 1987 TO 1990	June 25, 1990	36

16 SUPPLEMENTARY NOTATION

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Latent Trait Models, Mental Test Theory, Operating Characteristics, Nonparametric Estimation, Discrete Item Responses

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

A new procedure of nonparametric estimation of the operating characteristics of discrete item responses has been proposed, and it is called Differential Weight Procedure of the Conditional P.D.F. Approach. Some examples have been given, and sensitivities of the resulting estimated operating characteristics to irregularities of the differential weight functions have been observed and discussed. Usefulnesses of the method have also been discussed. These outcomes suggest the importance of further investigation of the weight function in the future.

20 DISTRIBUTION / AVAILABILITY OF ABSTRACT	21 ABSTRACT SECURITY CLASSIFICATION
☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	
22a NAME OF RESPONSIBLE INDIVIDUAL: Dr. Charles E. Davis	22b TELEPHONE (Include Area Code): 202-696-4046 / 22c OFFICE SYMBOL: ONR-1142-CS

DD Form 1473, JUN 86 — Previous editions are obsolete. — SECURITY CLASSIFICATION OF THIS PAGE

S/N 0102-LF-014-6603

TABLE OF CONTENTS

		Page
1	Introduction	1
2	Common Backgrounds and Differences among Different Procedures	1
3	Simple Sum Procedure of the Conditional P.D.F. Approach Combined with the Normal Approach Method	5
4	Differential Weight Procedure	5
5	Examples	7
6	Sensitivities to Irregularities of Weight Functions	16
7	Usefulnesses of the Differential Weight Procedure	24
8	Discussion and Conclusions	24

REFERENCES

The research was conducted at the principal investigator's laboratory, 405 Austin Peay Bldg., Department of Psychology, University of Tennessee, Knoxville, Tennessee. Those who worked as assistants for this research include Christine A. Golik, Barbara A. Livingston, Lee Hai Gan and Nancy H. Domm.

I Introduction

In the past couple of decades the author has been engaged in the nonparametric estimation of the operating characteristics of discrete item responses in the context of latent trait models (cf. Samejima, 1981b, 1988). As early as in 1977 the author proposed Normal Approximation Method (Samejima, 1977b) which can be used for the item calibration both in computerised adaptive testing and in paper-and-pencil testing. She also discussed the effective use of information functions in adaptive testing (Samejima, 1977a). Since then, with the support by the Office of Naval Research, she has developed several approaches and methods for the same purpose (cf. Samejima, 1977c, 1978a, 1978b, 1978c, 1978d, 1978e, 1978f, 1980a, 1980b, 1981a; Samejima and Changas, 1981). For convenience, they can be categorised as follows.

Approaches	Methods
(1) Bivariate P.D.F. Approach	(1) Pearson System Method
(2) Histogram Ratio Approach	(2) Two-Parameter Beta Method
(3) Curve Fitting Approach	(3) Normal Approach Method
(4) Conditional P.D.F. Approach	(4) Lognormal Approach Method
(4.1) Simple Sum Procedure	
(4.2) Weighted Sum Procedure	
(4.3) Proportioned Sum Procedure	

Here by an approach we mean a general procedure in approaching the operating characteristics of a discrete item response, and by a method we mean a specific method in approximating the conditional density of ability, given its maximum likelihood estimate. Thus a combination of an approach and a method provides us with a specific procedure for estimating the operating characteristic of a discrete item response.

These approaches and methods are characterised by two features, i.e.,

(1) estimation is made without assuming any mathematical forms for the operating characteristics of discrete item responses, and

(2) estimation is efficient enough to base itself upon a relatively small set of data of, say, several hundred to a few thousand examinees.

The present paper proposes a method which increases accuracies of estimation of the operating characteristics of discrete item responses, especially when the true operating characteristic is represented by a *steep* curve, and also at the lower and upper ends of the ability distribution where the estimation tends to be inaccurate because of smaller numbers of subjects involved in the base data. Tentatively, it is called the *Differential Weight Procedure*, and it belongs to the Conditional P.D.F. Approach. This procedure costs more CPU time than the Simple Sum Procedure, which has been used frequently (cf. Samejima, 1981b, 1988), but the advantage of handling more than one item, say, fifty, together in the Conditional P.D.F. Approach is still there.

II Common Backgrounds and Differences among Different Procedures

Let θ be ability, or latent trait, which assumes any real number. We assume that there is a set of test items measuring θ whose characteristics are known. This set of test items is called Old Test, whose meaning is somewhat close to the original itempool in the adaptive testing situation.

Let h denote an item of the Old Test, k_h be a discrete item response to item h, and $P_{k_h}(\theta)$ be the operating characteristic of k_h, or the conditional probability assigned to k_h, given θ. We assume that $P_{k_h}(\theta)$ is three-times differentiable with respect to θ. We have for the *item response information function*, $I_{k_h}(\theta)$, (Samejima, 1969, 1972)

$$I_{k_h}(\theta) = -\frac{\partial^2}{\partial\theta^2}\log P_{k_h}(\theta) \quad , \tag{2.1}$$

and the *item information function*, $I_h(\theta)$, is defined as the conditional expectation of $I_{k_h}(\theta)$, given θ, such that

$$I_h(\theta) = E[I_{k_h}(\theta) \mid \theta] = \sum_{k_h} I_{k_h}(\theta) P_{k_h}(\theta) \quad . \tag{2.2}$$

Let V be a response pattern such that

$$V = \{\, k_h \,\}' \qquad h = 1, 2, ..., n \quad . \tag{2.3}$$

The operating characteristic, $P_V(\theta)$, of the response patten V is defined as the conditional probability of V, given θ, and by virtue of *local independence* we can write

$$P_V(\theta) = \prod_{k_h \in V} P_{k_h}(\theta) \quad . \tag{2.4}$$

The *response pattern information function*, $I_V(\theta)$, is given by

$$I_V(\theta) = -\frac{\partial^2}{\partial\theta^2}\log P_V(\theta) = \sum_{k_h \in V} I_{k_h}(\theta) \quad , \tag{2.5}$$

and the test information function, $I(\theta)$, is defined as the conditional expectation of $I_V(\theta)$, given θ, and we obtain from (2.1), (2.2), (2.3), (2.4) and (2.5)

$$I(\theta) = E[I_V(\theta) \mid \theta] = \sum_V I_V(\theta) P_V(\theta) = \sum_{h=1}^{n} I_h(\theta) \quad . \tag{2.6}$$

For the sake of simplicity in handling mathematics, the tentative transformation of θ to τ is made by

$$\tau = C_1^{-1} \int_{-\infty}^{\theta} [I(t)]^{1/2} dt + C_0 \quad , \tag{2.7}$$

where C_0 is an arbitrary constant for adjusting the origin of τ, and C_1 is an arbitrary constant which equals the square root of the test information functions, $I^*(\tau)$, of τ, so that we can write

$$C_1 = [I^*(\tau)]^{1/2} \tag{2.8}$$

for all τ. This transformation will be simplified if we use a polynomial approximation to the square root of the test information function, $[I(\theta)]^{1/2}$, in the least squares sense which is accomplished by using the method of moments (cf. Samejima and Livingston, 1979) for the meaningful interval of τ. Thus (2.7) can be changed to the form

(2.9) $$\tau \doteq C_1^{-1} \sum_{k=0}^{m} \alpha_k (k+1)^{-1} \theta^{k+1} + C_0$$

$$= \sum_{k=0}^{m+1} \alpha_k^* \theta^k \ ,$$

where α_k $(k = 0, 1, \ldots, m)$ is the k-th coefficient of the polynomial of degree m approximating the square root of $I(\theta)$, and α_k^* is the new k-th coefficient which is given by

(2.10) $$\alpha_k^* \begin{cases} = C_0 & k = 0 \\ = (C_1 k)^{-1} \alpha_{k-1} & k = 1, 2, \ldots, m+1 \ . \end{cases}$$

With this transformation of θ to τ and by virtue of (2.8), we can use the asymptotic normality with the two parameters, τ and C_1^{-1}, as the approximation to the conditional distribution of the maximum likelihood estimator $\hat{\tau}$, given its true value τ (cf. Samejima, 1981b). Then the first through fourth conditional moments of τ, given $\hat{\tau}$, can be obtained from the density function, $g^*(\hat{\tau})$, of $\hat{\tau}$ and from the constant C_1 by the following four formulae (cf. Samejima, 1981b):

(2.11) $$E(\tau \mid \hat{\tau}) = \hat{\tau} + C_1^{-2} \frac{d}{d\hat{\tau}} \log g^*(\hat{\tau}) \ ,$$

(2.12) $$Var.(\tau \mid \hat{\tau}) = C_1^{-2} [1 + C_1^{-2} \frac{d^2}{d\hat{\tau}^2} \log g^*(\hat{\tau})] \ ,$$

(2.13) $$E[\{\tau - E(\tau \mid \hat{\tau})\}^3 \mid \hat{\tau}] = C_1^{-6} [\frac{d^3}{d\hat{\tau}^3} \log g^*(\hat{\tau})]$$

and

(2.14) $$E[\{\tau - E(\tau \mid \hat{\tau})\}^4 \mid \hat{\tau}] = C_1^{-4} [3 + 6C_1^{-2} \{\frac{d^2}{d\hat{\tau}^2} \log g^*(\hat{\tau})\} + 3C_1^{-4} \{\frac{d^2}{d\hat{\tau}^2} \log g^*(\hat{\tau})\}^2$$

$$+ C_1^{-4} \{\frac{d^4}{d\hat{\tau}^4} \log g^*(\hat{\tau})\}] \ .$$

This density function, $g^*(\hat{\tau})$, can be estimated by fitting a polynomial, using the method of moments (cf. Samejima and Livingston, 1979), as we did in the transformation of θ to τ, based upon the empirical set of $\hat{\tau}$'s. Note that in the above formulae the first moment is about the origin, while the other three are about the mean.

The two coefficients, β_1 and β_2, and Pearson's criterion κ are obtained by

(2.15) $$\beta_1 = \mu_3^2 \, \mu_2^{-3} \ ,$$

(2.16) $$\beta_2 = \mu_4 \, \mu_2^{-2}$$

and

$$\kappa = \beta_1(\beta_2+3)^2[4(2\beta_2-3\beta_1-6)(4\beta_2-3\beta_1)]^{-1} \quad , \tag{2.17}$$

by substituting μ_2 , μ_3 and μ_4 by $Var.(\tau \mid \hat{\tau})$, $E[\{\tau - E(\tau \mid \hat{\tau})\}^3 \mid \hat{\tau}]$ and $E[\{\tau - E(\tau \mid \hat{\tau})\}^4 \mid \hat{\tau}]$ respectively, which are obtained by formulae (2.12), (2.13) and (2.14).

In the Bivariate P.D.F. Approach, we approximate the bivariate distribution of the transformed latent trait τ and its maximum likelihood estimate $\hat{\tau}$ *for each subpopulation of examinees who share the same discrete item response* to a specified item. Thus the procedure must be repeated as many times as the number of discrete item response categories for each separate item. It is rather a time-consuming approach, and the CPU time for the item calibration increases almost proportionally to the number of *new* items.

In contrast to this, Conditional P.D.F. Approach deals with the total population of subjects, and all the items together. Effort is focused upon the approximation of the conditional distribution of τ , given $\hat{\tau}$, for the total population of examinees, and then the result is branched into separate discrete item response subpopulations for each item.

If we compare the two approaches with each other, therefore, we can say that Bivariate P.D.F. Approach is an orthodox approach, while Conditional P.D.F. Approach needs an assumption that the conditional distribution of τ , given $\hat{\tau}$, is unaffected by the different subpopulations of examinees. While this assumption can only be tolerated in most cases, the latter approach has two big advantages in the sense that the CPU time required in item calibration is substantially less, and that it does not have to deal with subgroups of small numbers of subjects in approximating the joint bivariate distributions of τ and $\hat{\tau}$.

In each of these two approaches, we can choose one of the four methods listed earlier in estimating the bivariate density of τ and $\hat{\tau}$, or the conditional density of τ , given its maximum likelihood estimate $\hat{\tau}$. In so doing, in the Pearson System Method, we use all four conditional moments of τ , given $\hat{\tau}$, which are estimated through the formulae (2.11) through (2.14), and, using Pearson's criterion κ , which is given by (2.17), one of the Pearson System density functions is selected. In the Two-Parameter Beta Method two of the four parameters of the Beta density function, i.e., the lower and upper endpoints of the interval of τ for which the Beta density is positive, are a priori given, and the other two parameters are estimated by using the first two conditional moments of τ , given $\hat{\tau}$, which are provided by (2.11) and (2.12), respectively. In the Normal Approach Method, again we use only the first two conditional moments of τ , given $\hat{\tau}$, as the first and second parameters of the normal density function.

If we compare these three methods, it will be appropriate to say that both Two-Parameter Beta Method and Normal Approach Method are simpler versions of Pearson System Method. And yet the latter two methods have an advantage of using only the first two estimated conditional moments of τ , given $\hat{\tau}$, whereas the former requires the additional third and fourth conditional moments, whose estimations are less accurate compared with those of the first two conditional moments. If we compare the Two-Parameter Beta Method with the Normal Approach Method, we will notice that the former allows non-symmetric density functions, while the latter does not. This is an advantage of the Two-Parameter Beta Method over the Normal Approach Method, and yet the former has the disadvantage of the requirement that two of the four parameters should a priori be set.

Lognormal Approach Method was developed later, which uses up to the third conditional moment and allows more flexibilities in the shape of the conditional distribution of τ , given $\hat{\tau}$, than the Normal Approach Method. It was intended that a *happy medium* between the Pearson System Method and the Normal Approach Method would be realised, in the effort of ameliorating the disadvantages of these

two methods and of keeping their separate advantages.

III Simple Sum Procedure of the Conditional P.D.F. Approach Combined with the Normal Approach Method

It is obvious from the discussion given in the preceding section that the Conditional P.D.F. Approach combined with the Normal Approach Method is the simplest and one of the most economical procedures in CPU time. Out of the three procedures of the Conditional P.D.F. Approach the Simple Sum Procedure is the simplest one (cf. Samejima, 1981b). For this reason, the combination of the Simple Sum Procedure of the Conditional P.D.F. Approach and the Normal Approach Method has most frequently been applied for simulated and empirical data. Fortunately, in spite of the simplicity of the procedure, the results with simulated data in the adaptive testing situation and with simulated and empirical data in the paper-and-pencil testing situation indicate that we can estimate the operating characteristics fairly accurately by using this combination (cf. Samejima, 1981b, 1984). This seems to prove the *robustness* of the Conditional P.D.F. Approach. For one thing, there is a good reason why Normal Approach Method works well, for the conditional distribution of τ, given $\hat{\theta}$, is indeed normal if the (unconditional) distribution of τ is normal, and it is a truncated normal distribution if the (unconditional) distribution of τ is rectangular, and the truncation is negligible for most of the conditional distributions.

In the Simple Sum Procedure of the Conditional P.D.F. Approach, the operating characteristic, $P_{k_g}(\theta)$, of the discrete item response k_g of an *unknown* item g is estimated through the formula

$$\hat{P}_{k_g}(\theta) = \hat{P}^*_{k_g}[\tau(\theta)] = \sum_{s \in k_g} \phi(\tau \mid \hat{\theta}_s)[\sum_{s=1}^{N} \phi(\tau \mid \hat{\theta}_s)]^{-1} \quad , \tag{3.1}$$

where $s\ (= 1, 2, \ldots, N)$ indicates an individual examinee, and $\phi(\tau \mid \hat{\theta}_s)$ denotes the conditional density of τ, given $\hat{\theta}_s$. This conditional density is estimated by using the estimated conditional moments of τ, given $\hat{\theta}_s$, using one of the four methods, as was described in the preceding section.

In the Weighted Sum Procedure of the Conditional P.D.F. Approach, we have for the estimated operating characteristic of k_g

$$\hat{P}_{k_g}(\theta) = \hat{P}^*_{k_g}(\tau(\theta)) = \sum_{s \in k_g} w(\hat{\theta}_s)\phi(\tau \mid \hat{\theta}_s)[\sum_{s=1}^{N} w(\hat{\theta}_s)\phi(\tau \mid \hat{\theta}_s)]^{-1} \tag{3.2}$$

where $w(\hat{\theta}_s)$ is the weight function of $\hat{\theta}_s$. When we combine one of these two approaches with the Normal Approach Method, $\phi(\tau \mid \hat{\theta}_s)$ in (3.1) or in (3.2) is approximated by the normal density function, using the first two estimated conditional moments of τ, given $\hat{\theta}_s$, which are given by (2.11) and (2.12), respectively, as its parameters, $\mu_{\hat{\theta}_s}$ and $\sigma_{\hat{\theta}_s}$, in the formula

$$\phi(\tau \mid \hat{\theta}_s) = [2\pi]^{-1/2}[\sigma_{\hat{\theta}_s}]^{-1} \exp[-(\tau - \mu_{\hat{\theta}_s})^2/\{2\sigma^2_{\hat{\theta}_s}\}] \quad . \tag{3.3}$$

IV Differential Weight Procedure

If we accept the approximation of the conditional distribution of $\hat{\theta}$, given τ, by the asymptotic normality, as we do in these approaches (cf. Samejima, 1981b), the other conditional distribution, i.e., that of τ, given $\hat{\theta}$, will become more or less *incidental*. Thus in the Bivariate P.D.F. Approach the bivariate distribution of τ and $\hat{\theta}$ is approximated for each separate item score subpopulation of subjects of each unknown test item. In the Conditional P.D.F. Approach, however, the incidentality

of this second conditional distribution is not rigorously considered, and the implicit assumption exists such that for the fixed value of $\hat{\tau}$ the conditional distributions of τ are similar for the different item score subpopulations.

Take the dichotomous response level, for example. On this level, each item is scored "right" or "wrong", "affirmative" or "negative", etc. The above assumption of non-incidentality may be acceptable when the operating characteristic of the correct answer of the item is represented by a *mildly steep* curve, as is the case with most practical situations, and the questions are asked to subjects whose ability levels are *compatible* with the difficulty levels of the questions, as is the case with adaptive testing and, though less rigorously, with many cases of paper-and-pencil testing.

This assumption is not acceptable, however, when the operating characteristic of the correct answer is represented by a *steep* curve. If the operating characteristic follows the Guttman scale, for example, then the conditional distributions of τ, given $\hat{\tau}$, for the two separate item score subpopulations are distinctly separated, and they do not even overlap! If we use the Simple Sum Procedure or the Weighted Sum Procedure for an item which nearly follows the Guttman scale, therefore, the resulting estimated operating characteristics of the correct and the incorrect answers will tend to be *flatter* than they actually are.

This problem can be solved by estimating *differential* conditional distributions of τ, given $\hat{\tau}$, for the separate discrete item responses to an "unknown" item. Let $\phi_{k_g}(\tau \mid \hat{\tau})$ denote the conditional density of τ, given $\hat{\tau}$, for the subpopulation of subjects who share the same discrete item response k_g to an "unknown" item g. We can write

$$\phi_{k_g}(\tau \mid \hat{\tau}) = f^*_{k_g}(\tau)\ \psi(\hat{\tau} \mid \tau)\ [g^*_{k_g}(\hat{\tau})]^{-1} \quad , \tag{4.1}$$

where $f^*_{k_g}(\tau)$ indicates the density of τ for the subpopulation of subjects who share k_g as their common item score of item g, $\psi(\hat{\tau} \mid \tau)$ is the conditional density of $\hat{\tau}$, given τ, which is approximated by the normal density, $n[\tau,\ C_1^{-1}]$, and $g^*_{k_g}(\hat{\tau})$ is the marginal density of $\hat{\tau}$, for this subpopulation, and for which we have

$$g^*_{k_g}(\hat{\tau}) = \int_{-\infty}^{\infty} f^*_{k_g}(\tau)\ \psi(\hat{\tau} \mid \tau)\ d\tau \quad . \tag{4.2}$$

We notice that there is a relationship

$$f^*_{k_g}(\tau) = f^*(\tau)\ P^*_{k_g}(\tau)\ [\int_{-\infty}^{\infty} f^*(\tau)\ P^*_{k_g}(\tau)\ d\tau]^{-1} \quad , \tag{4.3}$$

where $f^*(\tau)$ denotes the density of τ for the total population. Since we have

$$\phi(\tau \mid \hat{\tau}) = f^*(\tau)\ \psi(\hat{\tau} \mid \tau)\ [g^*(\hat{\tau})]^{-1} \quad , \tag{4.4}$$

where $g^*(\hat{\tau})$ is the density of $\hat{\tau}$ for the total population of subjects, as was described in the preceding section, which is given by

$$g^*(\hat{\tau}) = \int_{-\infty}^{\infty} f^*(\tau)\ \psi(\hat{\tau} \mid \tau)\ d\tau \quad , \tag{4.5}$$

from the above formulae we obtain

(4.6) $$\phi_{k_g}(\tau \mid \hat{\theta}) = \phi(\tau \mid \hat{\theta})\ P^*_{k_g}(\tau)\ h(\hat{\theta}) \quad ,$$

where $h(\hat{\theta})$ is a function of $\hat{\theta}$ and constant for a fixed value of $\hat{\theta}$. Thus $\phi_{k_g}(\tau \mid \hat{\theta})$ is a density function proportional to $\phi(\tau \mid \hat{\theta})\ P^*_{k_g}(\tau)$. We notice that $\phi(\tau \mid \hat{\theta})$ in this formula is common to all the item scores and across different unknown items, while $P^*_{k_g}(\tau)$ is a specific function of τ for each k_g. Since $\phi(\tau \mid \hat{\theta})$ can be estimated by one of the four methods described earlier, our effort should be focused on finding an appropriate differential weight function for each k_g. Let $W_{k_g}(\tau)$ denote such a *differential weight function*, which replaces $P^*_{k_g}(\tau)\ h(\hat{\theta})$ in (4.6). Thus we can revise (3.1) and (3.2) into the forms

(4.7) $$\hat{P}_{k_g}(\theta) = \hat{P}^*_{k_g}[\tau(\theta)] = \sum_{s \in k_g} W_{k_g}(\tau)\phi(\tau \mid \hat{\theta}_s)[\sum_{s=1}^{N} W_{k_g}(\tau; s)\phi(\tau \mid \hat{\theta}_s)]^{-1}$$

and

(4.8) $$\hat{P}_{k_g}(\theta) = \hat{P}^*_{k_g}(\tau(\theta)) = \sum_{s \in k_g} w(\hat{\theta}_s)W_{k_g}(\tau)\phi(\tau \mid \hat{\theta}_s)[\sum_{s=1}^{N} w(\hat{\theta}_s)W_{k_g}(\tau; s)\phi(\tau \mid \hat{\theta}_s)]^{-1} \quad .$$

Since the differential weight function $W_{k_g}(\tau)$ involves $P^*_{k_g}(\tau)$, which itself is the target of estimation, we may use its estimate, $\hat{P}^*_{k_g}(\tau)$, obtained by the Simple Sum Procedure or by the Weighted Sum Procedure, as its substitute. In so doing we may need some local smoothings of $\hat{P}^*_{k_g}(\tau)$ where the estimation involves substantial amounts of error because of locally small numbers of subjects in the base data, etc. In some cases we may need several iterations by renewing the differential weight functions on each stage until the resulting estimated operating characteristic converges.

V Examples

We have tried this proposed method on the simulated data provided by Dr. Charles Davis of the Office of Naval Research, using the Simple Sum Procedure of the Conditional P.D.F. Approach combined with the Normal Approach Method with some modifications as the initial estimate of $P_{k_g}(\tau)$ in the differential weight function. These data are simulated on-line item calibration data of the initial itempool calibration based upon conventional testing, in which 100 dichotomous items are divided into four subtests of 25 items each, and each subtest has been administered to 6,000 hypothetical examinees, and those of different rounds based upon adaptive testing, in which each of the 50 new binary items has been administered to a subgroup of 1,500 hypothetical subjects out of the total of 15,000. These hypothetical examinees' ability distributes unimodally within the interval of θ, (-3.0,3.0), with slight negative skewness.

For the purpose of illustration, Figure 5-1 presents the results of the Differential Weight Procedure using the results of the Simple Sum Procedure of the Conditional P.D.F. Approach combined with the Normal Approach Method with some modifications as the initial estimates, for eight items of the initial itempool. They are dichotomous items, and were intentionally selected from those items whose true operating characteristics of the correct answer are non-monotonic, in order to visualise the benefit of the nonparametric estimation of the operating characteristic. In each graph, also presented for comparison is the best fitted operating characteristic of the correct answer following the three-parameter logistic model, which has been given by Dr. Michael Levine. The logistic model on the dichotomous level is represented by

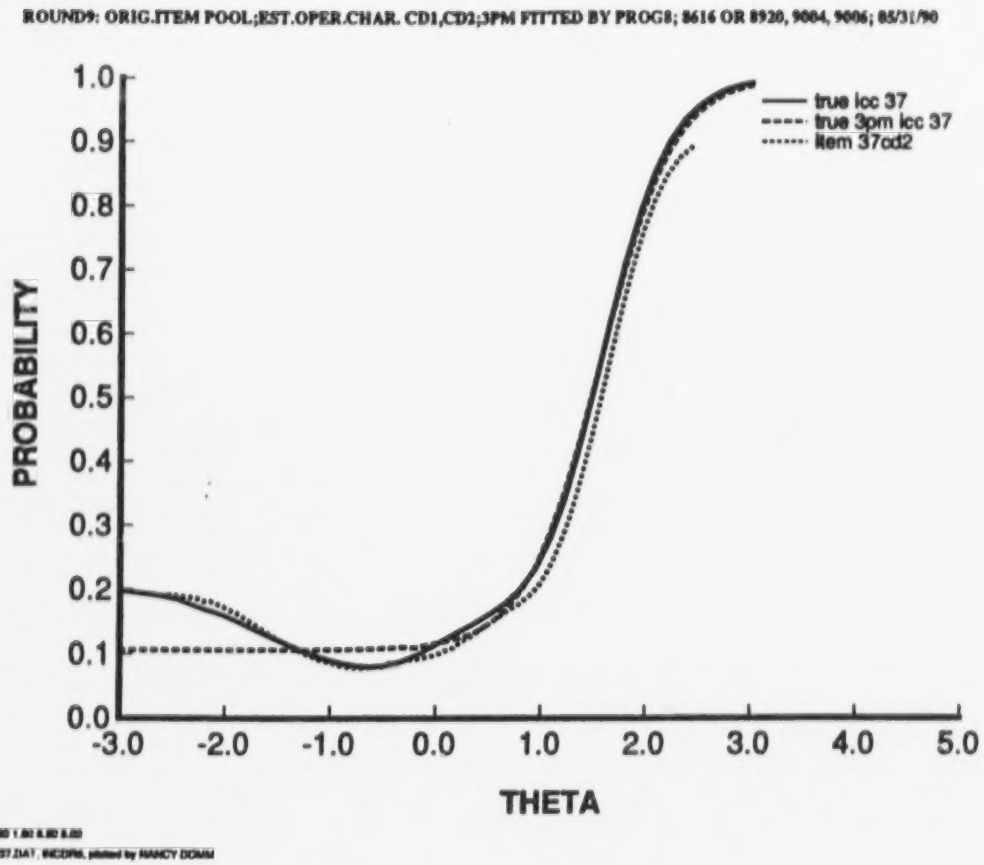

FIGURE 5-1

Eight Examples of the Estimated Operating Characteristic of the Correct Answer Using the Differential Weight Procedure (Dotted Line), in Comparison with the True Operating Characteristic (Solid Line) and the Best Fitted Three-Parameter Logistic Curve (Dashed Line).

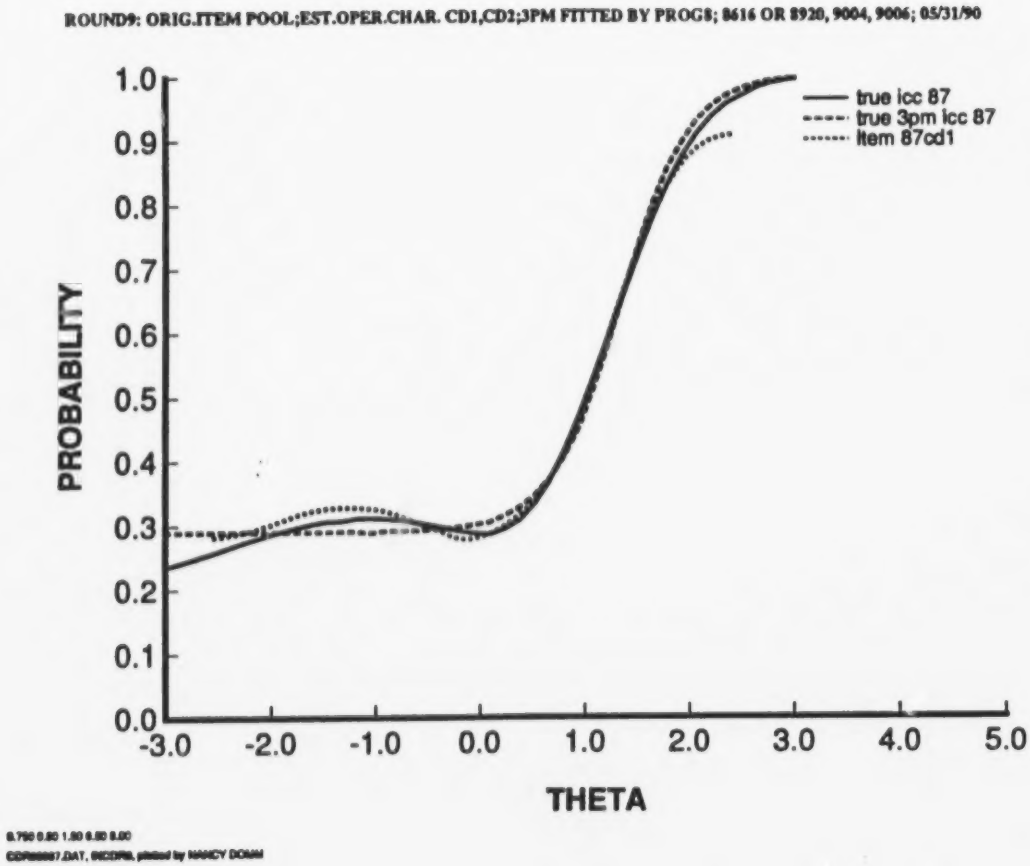

FIGURE 5-1 (Continued)

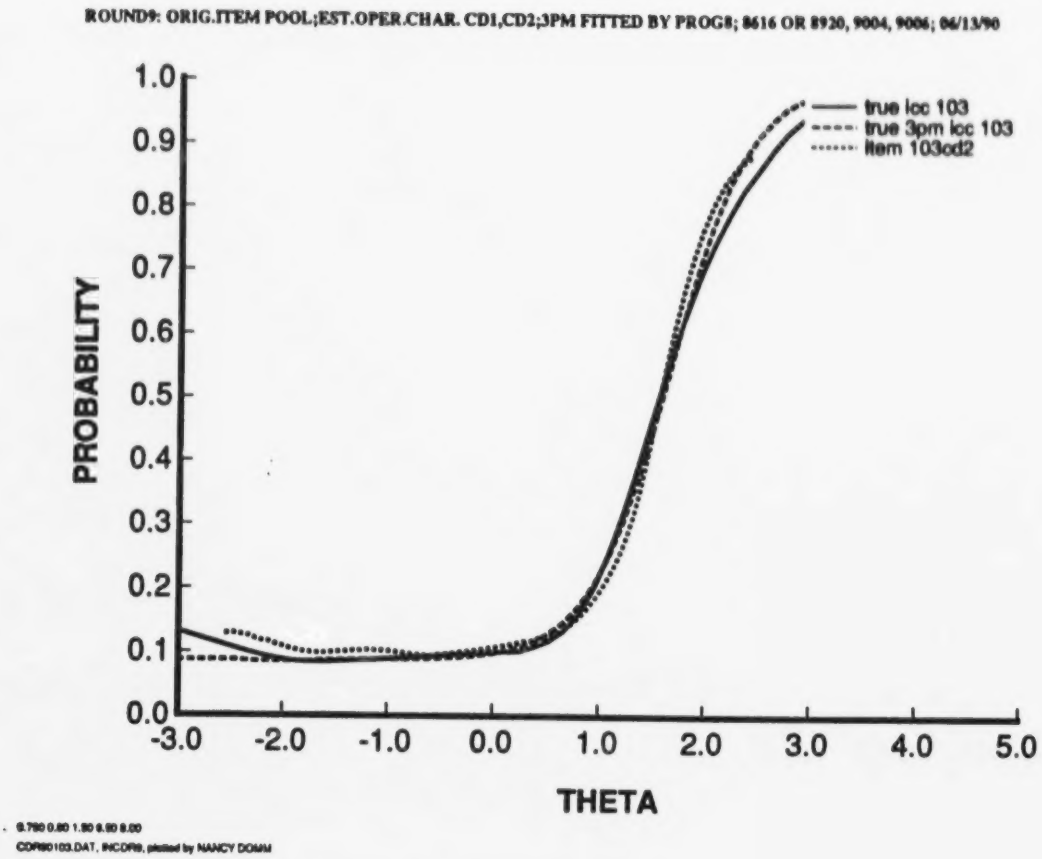

FIGURE 5-1 (Continued)

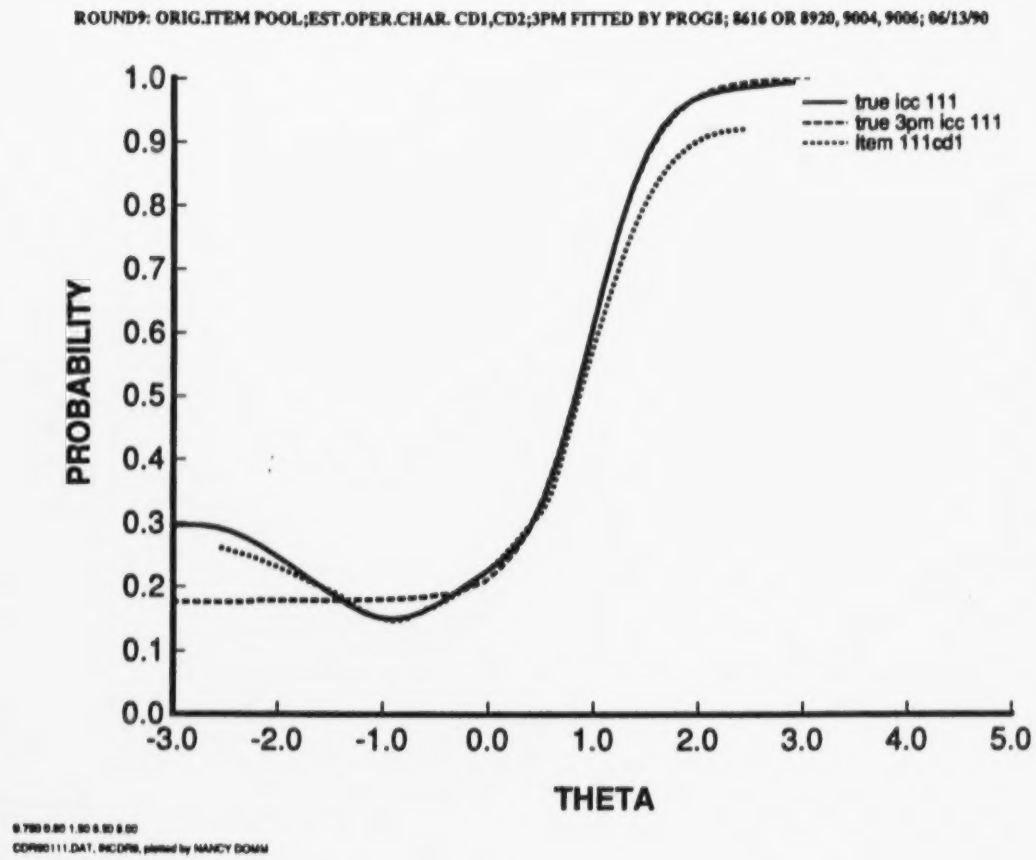

FIGURE 5-1 (Continued)

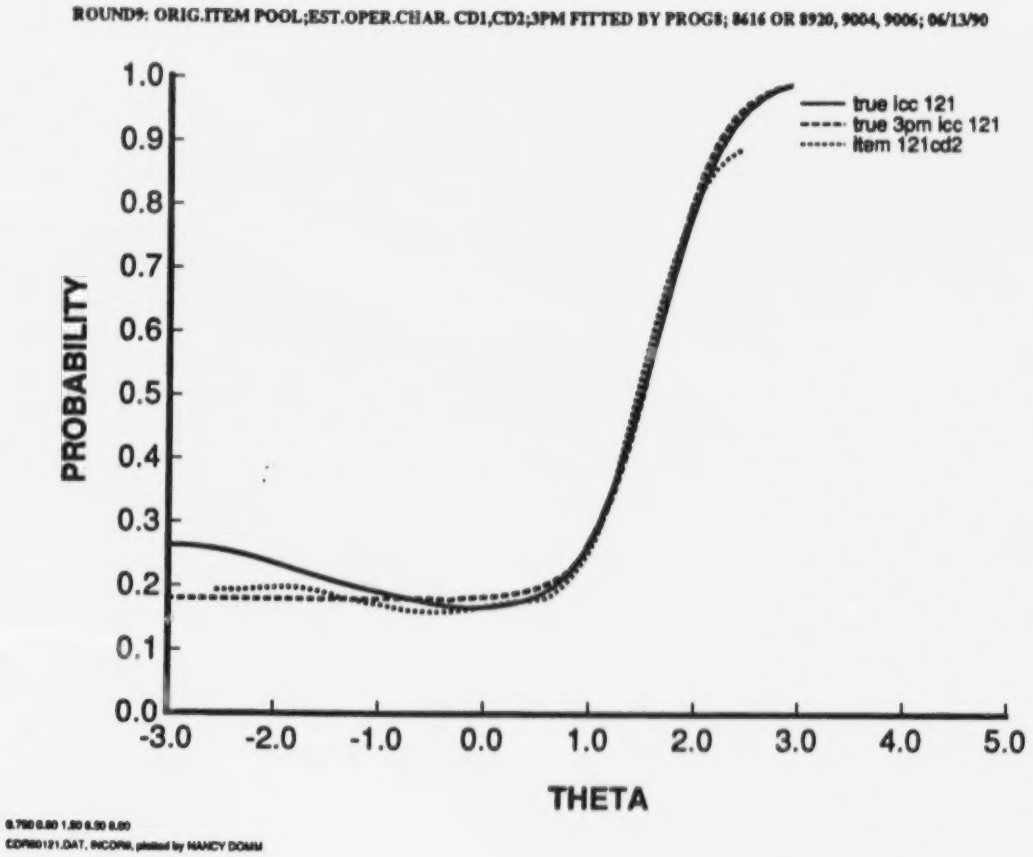

FIGURE 5-1 (Continued)

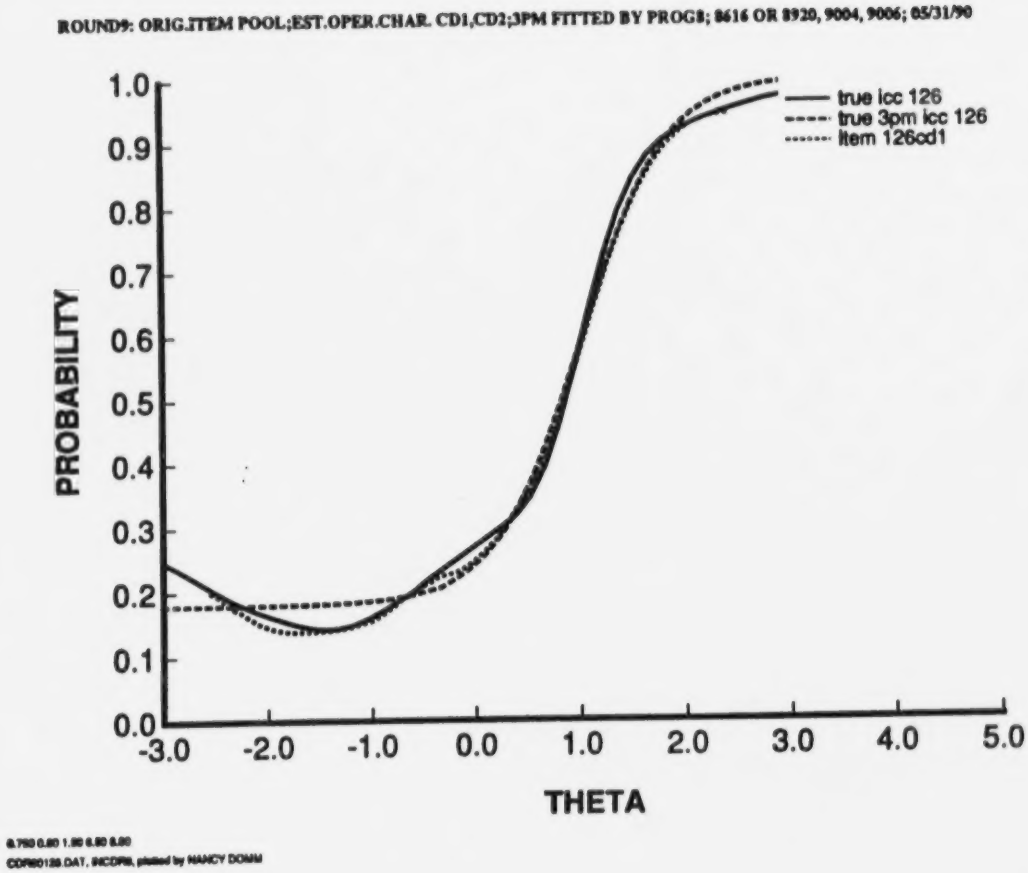

FIGURE 5-1 (Continued)

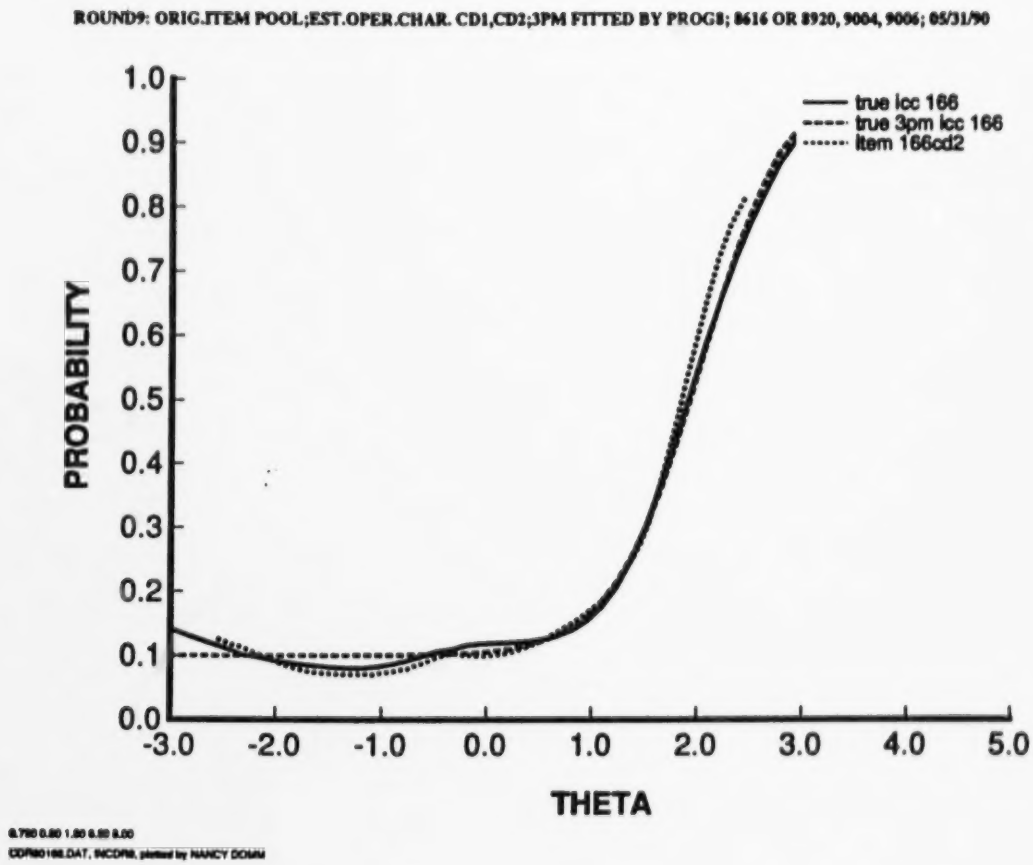

FIGURE 5-1 (Continued)

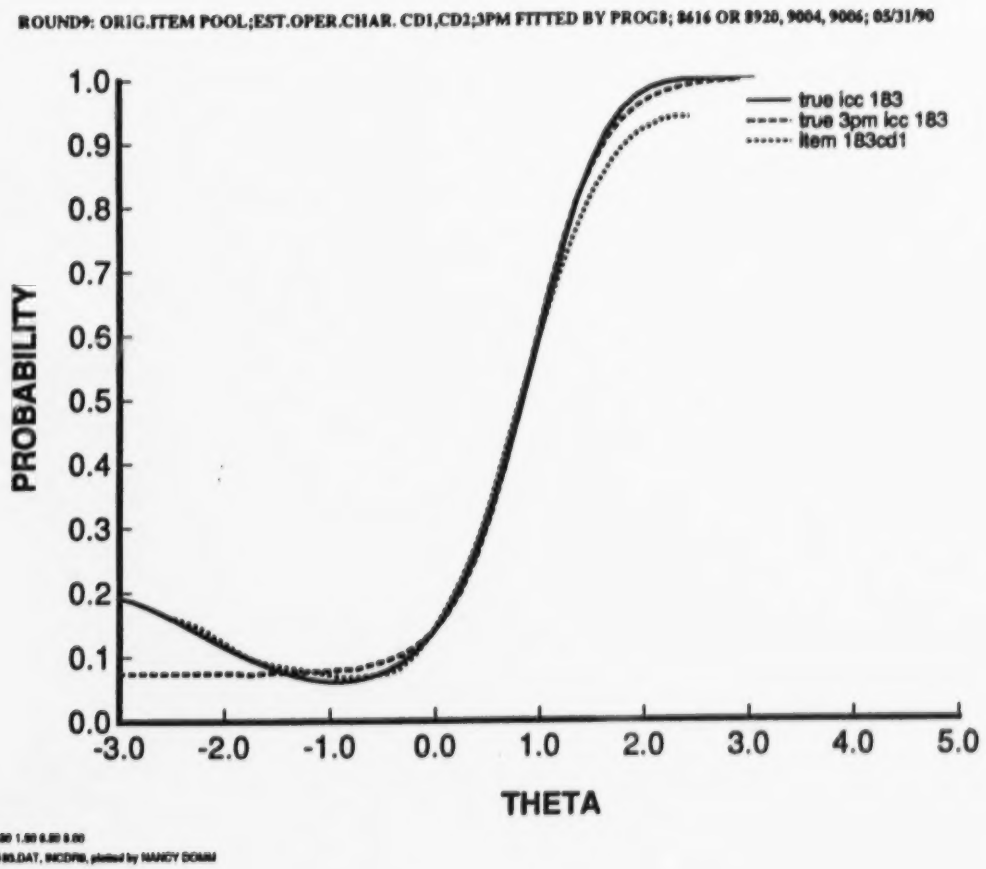

FIGURE 5-1 (Continued)

(5.1) $$P_g(\theta) = [1 + \exp\{-Da_g(\theta - b_g)\}]^{-1} \quad ,$$

where $P_g(\theta)$ denotes the operating characteristic of the correct answer to item g, a_g and b_g are the item discrimination and difficulty parameters, respectively, and D is a scaling factor which is usually set equal to 1.7. We can see in these graphs that the resulting estimated operating characteristics are fairly close to the true ones, and that they reflect the non-monotonicities.

VI Sensitivities to Irregularities of Weight Functions

As we have proceeded, several factors have been identified and observed which affect the resulting estimated operating characteristics substantially. They are concerned with the differential weight function, and can be itemised as: 1) lower end ambiguities, 2) upper end ambiguities, 3) local irregularities and 4) overall irregularities.

Out of these factors, lower and upper end ambiguities basically come from the fact that we do not usually have sufficiently large numbers of subjects on the lowest and the highest ends of the interval of θ of interest upon which the estimation of the operating characteristics is made. Also the fact that the test information function $I(\theta)$ is used in the transformation of θ to τ which is specified by (2.7) may have something to do with these ambiguities. It has been observed (Samejima, 1979b) that in using equivalent items following the Constant Information Model (Samejima, 1979a) the speed of convergence of the conditional distribution of the maximum likelihood estimate $\hat{\theta}$, given θ, to the asymptotic normality with θ and $[I(\theta)]^{-1/2}$ as its two parameters substantially differs for different levels of θ, *in spite of the fact that the amount of test information is constant for every level of* θ. To be more specific, the convergence is observed to be much slower at those levels which are close to either end of the interval of θ for which the amount of test information is non-zero and constant, and faster at intermediate levels of θ. This situation can be ameliorated if we replace the test information function $I(\theta)$ in (2.7) by one of its two modified forms (cf. Samejima, 1990a). We can write for the Modification Formula No. 1, $\Upsilon(\theta)$,

(6.1) $$\Upsilon(\theta) = I(\theta)\,[1 + \frac{\partial}{\partial\theta} B(\hat{\theta}_V \mid \theta)]^{-2} \quad ,$$

which is the reciprocal of an approximate minimum bound of the *variance* of the maximum likelihood estimator, where $B(\hat{\theta}_V \mid \theta)$ is the MLE bias function of the test consisting of items with any discrete item responses k_h. In the general case of discrete item responses, we can write for the bias function of the maximum likelihood estimate

(6.2) $$B(\hat{\theta}_V \mid \theta) = E[\hat{\theta}_V - \theta \mid \theta] = -(1/2)[I(\theta)]^{-2} \sum_{h=1}^{n} \sum_{k_h} P'_{k_h}(\theta) P''_{k_h}(\theta) [P_{k_h}(\theta)]^{-1} \quad ,$$

where, as before, $P_{k_h}(\theta)$ is the operating characteristic of the discrete response k_h, and $P'_{k_h}(\theta)$ and $P''_{k_h}(\theta)$ denote the first and second partial derivatives of $P_{k_h}(\theta)$ with respect to θ, respectively. Modification Formula No. 2, $\Xi(\theta)$, is given by

(6.3) $$\Xi(\theta) = I(\theta)\,\{[1 + \frac{\partial}{\partial\theta} B(\hat{\theta}_V \mid \theta)]^2 + I(\theta)\,[B(\hat{\theta}_V \mid \theta)]^2\}^{-1} \quad ,$$

which is the reciprocal of an approximate minimum bound of the *mean squared error* of the maximum likelihood estimator. When the MLE bias function of the test is *monotone increasing*, as is the case in many situations, it is obvious from (6.1), (6.2) and (6.3) that we have the relationship,

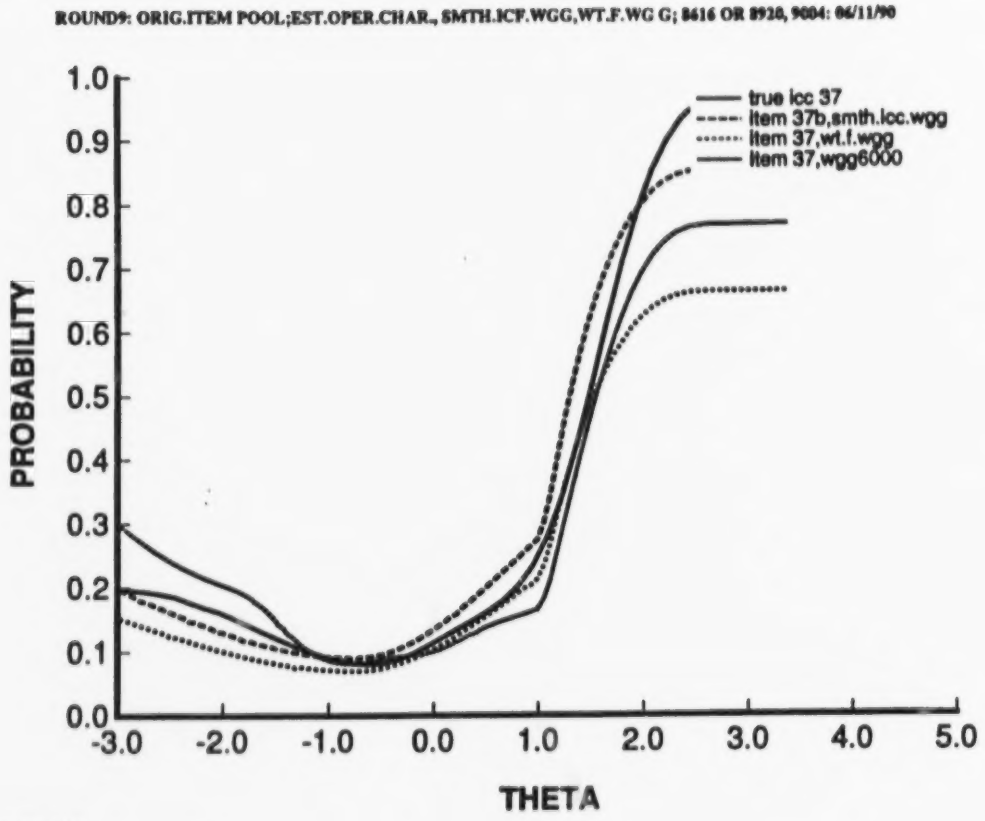

FIGURE 6-1

Seven Examples of the Estimated Operating Characteristic of the Correct Answer Using the Differential Weight Procedure (Dotted Line), in Comparison with the True Operating Characteristic (Solid Line), When the Differential Weight Function (Short Dashed Line) Has Irregularities. The Function Was Also Proportionally Enlarged and Plotted (Long Dashed Line) to Visualise the Angles and Other Irregularities Well.

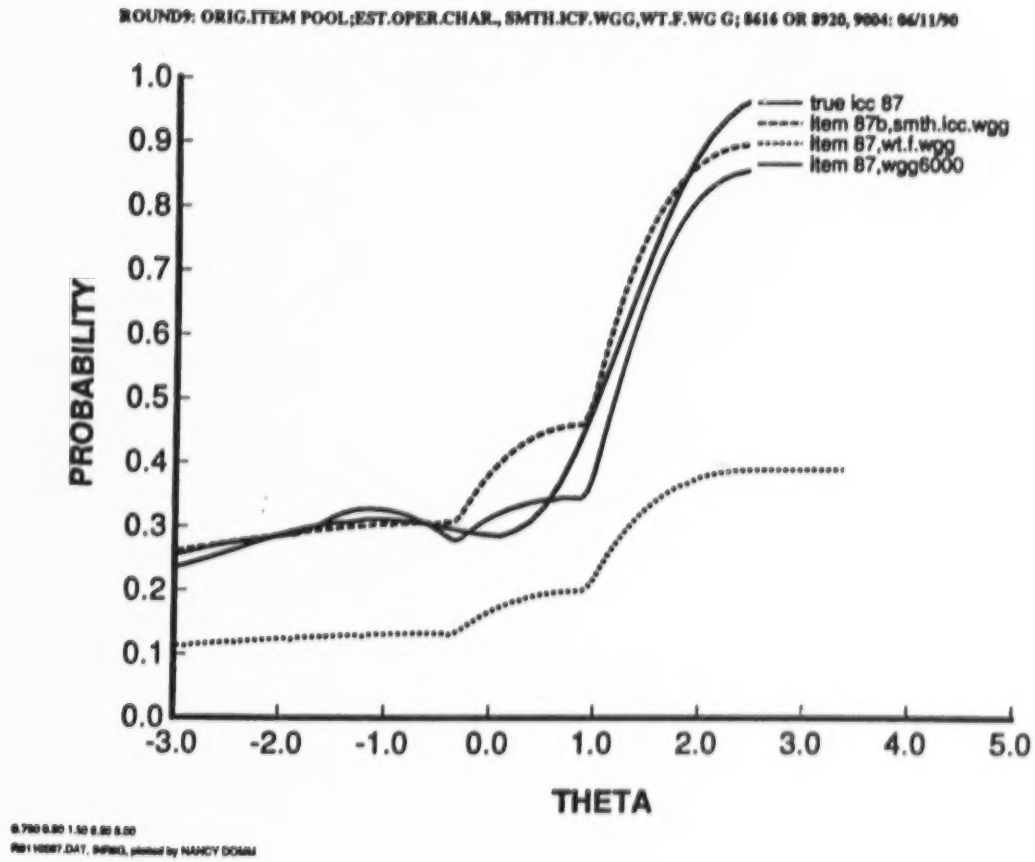

FIGURE 6-1 (Continued)

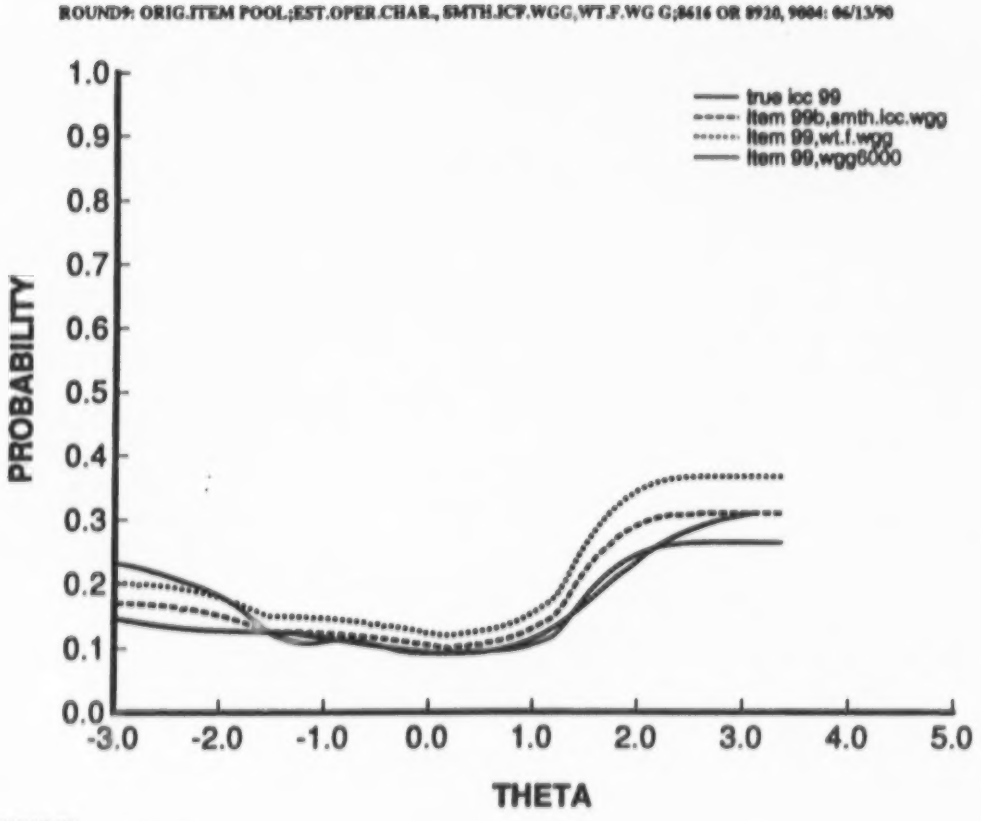

FIGURE 6-1 (Continued)

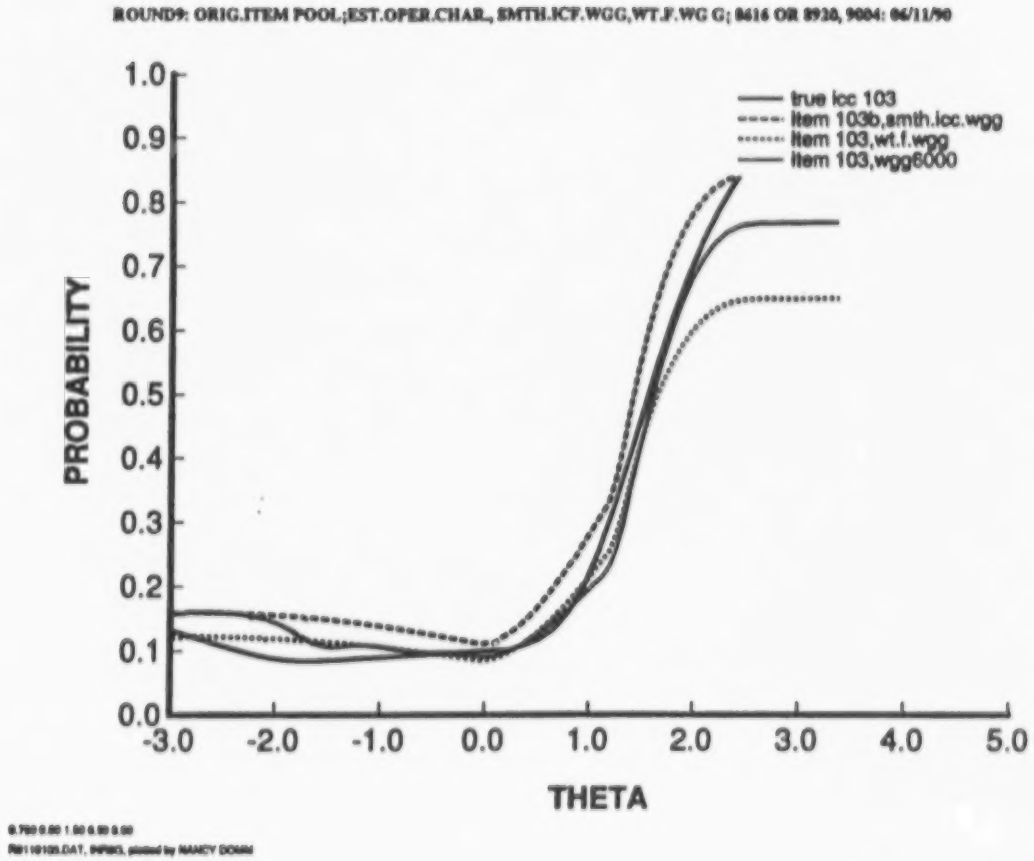

FIGURE 6-1 (Continued)

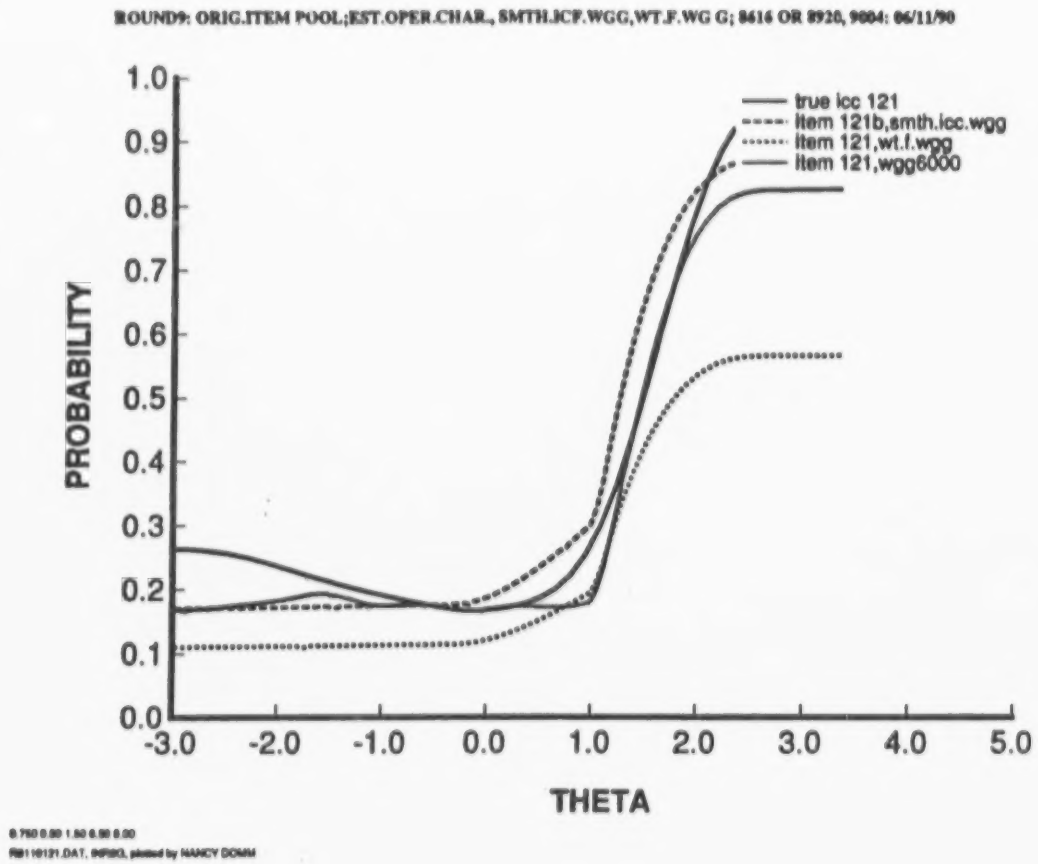

FIGURE 6-1 (Continued)

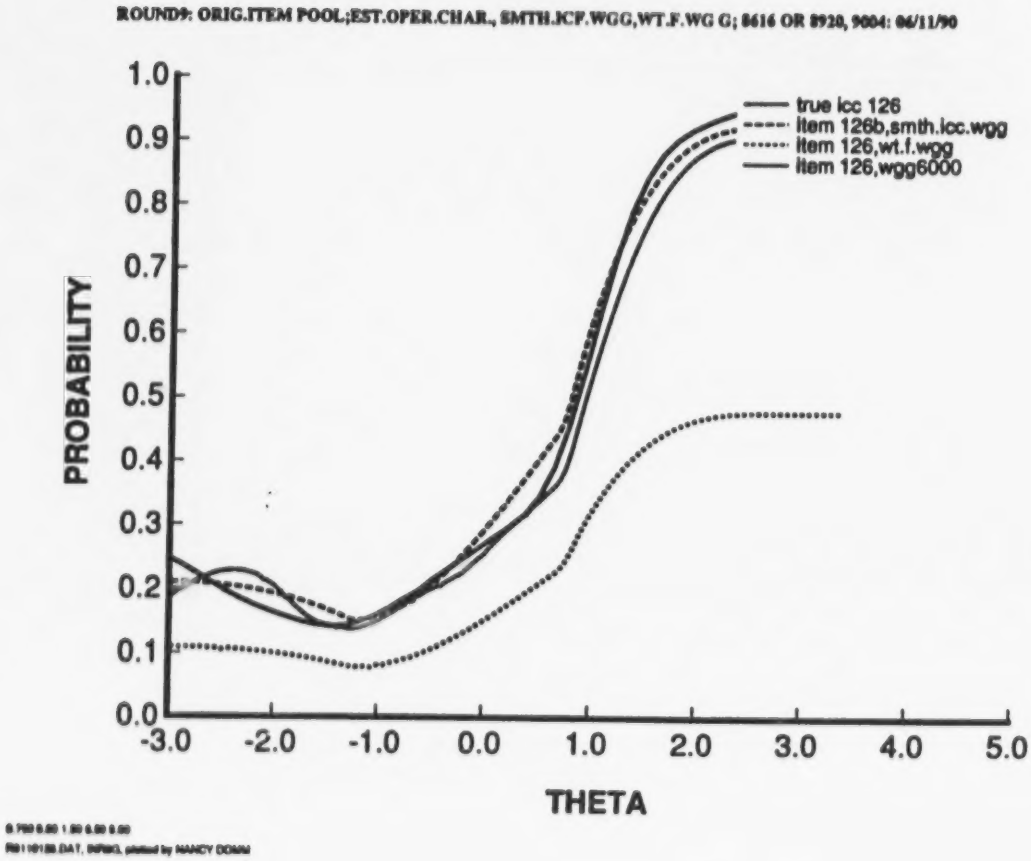

FIGURE 6-1 (Continued)

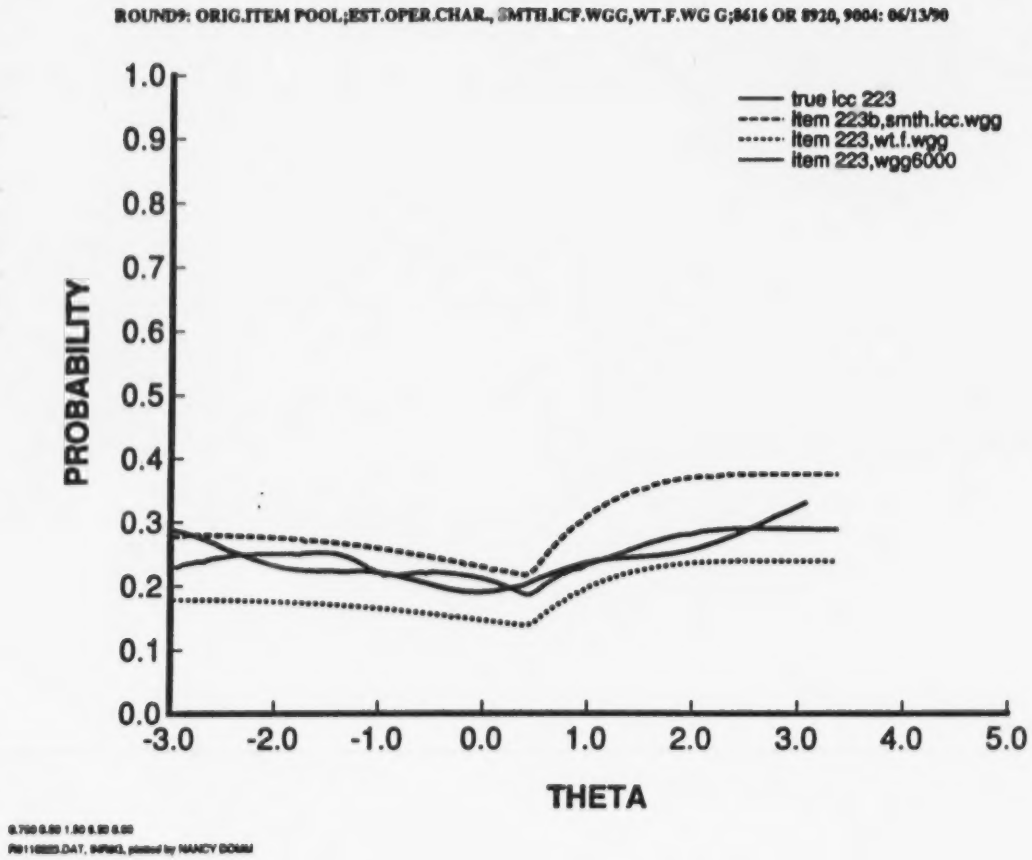

FIGURE 6-1 (Continued)

(6.4) $$E(\theta) \leq T(\theta) \leq I(\theta) \ ,$$

where the first inequality in (6.4) always holds regardless of the shape of the MLE bias function. Which one of the two modified test information functions is more appropriate to use depends upon the situation, and we need more investigation to answer this question.

By irregularity we mean non-smoothness, which is exemplified by an unnatural angle, etc. It has been observed that for most items the resulting operating characteristic is amazingly sensitive to these irregularities of the differential weight function. In order to observe these sensitivities, Figure 6-1 illustrates how these irregularities, which are involved in the differential weight function, affect the resulting estimated operating characteristic.

The effect of *local* irregularities is most interesting to observe in these examples presented by Figure 6-1. In each of these graphs, the artificially *irregular* differential weight function for the correct answer is drawn by a short dashed line, and, in order to emphasize its irregularities, it was proportionally enlarged and shown by a long dashed line. We can see in each graph that, when the differential weight function has an unnatural angle, for example, the resulting estimated operating characteristic of the correct answer also shows an unnatural angle at approximately the same level of θ . We can also see in these graphs how *overall* irregularities of the differential weight function affect the resulting estimated operating characteristic, and how sensitive the latter is to the former. This type of sensitivity of the resulting estimated operating characteristic to the irregularities of the differential weight function is encouraging as well as threatening, for it promises success in the estimation provided that we succeed in finding the right differential weight function.

VII Usefulnesses of the Differential Weight Procedure

It is obvious that *item analysis* in the true sense of the word starts from the accurate estimation of the operating characteristics of the item responses. Thus the nonparametric estimation of the operating characteristic offers a great deal of information about an item, when it is successful. In this sense we can say that the Differential Weight Procedure provides us with promise for the successful item analysis in general.

Following this, we can conceive of many applications of the method for different purposes. To give some examples, it will be especially useful for the on-line item calibration in computerized adaptive testing; also it will be useful in the revision of multiple-choice test items in order to reduce the effect of noise and to make the ability estimation efficient (cf. Samejima, 1990b).

VIII Discussion and Conclusions

A new procedure of nonparametric estimation of the operating characteristics of discrete item responses has been proposed, and it is called Differential Weight Procedure of the Conditional P.D.F. Approach. Some examples have been given, and sensitivities of the resulting estimated operating characteristics to irregularities of the differential weight functions have been observed and discussed. Usefulnesses of the method have also been discussed.

These outcomes suggest the importance of further investigation of the weight function in the future.

To summarize, although Simple Sum Procedure of the Conditional P.D.F. Approach combined with the Normal Approach Method works reasonably well for the on-line item calibration of adaptive testing, and also for the paper-and-pencil testing, especially when the number of subjects is large, if we wish to increase the accuracy of estimation we can use the Differential Weight Procedure. The disadvantage

will be the added CPU time, so we need to consider the balance of the cost and accuracy of estimation before we make our decision. It will be less expensive, however, if we compare the CPU time required for the present procedure with the time required for the Bivariate P.D.F. Approach.

References

[1] Samejima, F. Estimation of latent ability using a response pattern of graded item scores. *Psychometrika Monograph, 16*, 1969.

[2] Samejima, F. A general model for free-response data. *Psychometrika Monograph, 17*, 1972.

[3] Samejima, F. A use of the information function in tailored testing. *Applied Psychological Measurement, 1*, 1977a, 233-247.

[4] Samejima, F. A method of estimating item characteristic functions using the maximum likelihood estimate of ability. *Psychometrika, 42*, 1977b, 163-191.

[5] Samejima, F. Estimation of the operating characteristics of item response categories I: Introduction to the Two-Parameter Beta Method. *ONR/RR-77-1*, 1977c.

[6] Samejima, F. Estimation of the operating characteristics of item response categories II: Further development of the Two-Parameter Beta Method. *ONR/RR-78-1*, 1978a.

[7] Samejima, F. Estimation of the operating characteristics of item response categories III: The Normal Approach Method and the Pearson System Method. *ONR/RR-78-2*, 1978b.

[8] Samejima, F. Estimation of the operating characteristics of item response categories IV: Comparison of the different methods. *ONR/RR-78-3*, 1978c.

[9] Samejima, F. Estimation of the operating characteristics of item response categories V: Weighted Sum Procedure in the Conditional P.D.F. Approach. *ONR/RR-78-4*, 1978d.

[10] Samejima, F. Estimation of the operating characteristics of item response categories VI: Proportioned Sum Procedure in the Conditional P.D.F. Approach. *ONR/RR-78-5*, 1978e.

[11] Samejima, F. Estimation of the operating characteristics of item response categories VII: Bivariate P.D.F. Approach with Normal Approach Method. *ONR/RR-78-6*, 1978f.

[12] Samejima, F. Constant information model: A new, promising item characteristic function. *ONR/RR-79-1*, 1979a.

[13] Samejima, F. Convergence of the conditional distribution of the maximum likelihood estimate, given latent trait, to the asymptotic normality: Observations made through the constant information model. *ONR/RR-79-3*, 1979b.

[14] Samejima, F. Estimation of the operating characteristics when the test information of the Old Test is not constant I: Rationale. *ONR/RR-80-2*, 1980a.

[15] Samejima, F. Estimation of the operating characteristics when the test information of the Old Test is not constant II: Simple Sum Procedure of the Conditional P.D.F. Approach/Normal Approach Method using three subtests of the Old Test. *ONR/RR-80-4*, 1980b.

[16] Samejima, F. Estimation of the operating characteristics when the test information of the Old Test is not constant II: Simple Sum Procedure of the Conditional P.D.F. Approach/Normal Approach Method using three subtests of the Old Test, No. 2. *ONR/RR-81-2*, 1981a.

[17] Samejima, F. Final Report: Efficient methods of estimating the operating characteristics of item response categories and challenge to a new model for the multiple-choice item. *Final Report of N00014-77-C-0360*, Office of Naval Research, 1981b.

[18] Samejima, F. Plausibility functions of Iowa Vocabulary Test Items estimated by the Simple Sum Procedure of the Conditional P.D.F. Approach. *ONR/RR-84-1*, 1984.

[19] Samejima, F. Final Report: Advancement of latent trait theory. *Final Report of N00014-81-C-0569*, Office of Naval Research, 1988.

[20] Samejima, F. Modification of the test information fucntion. *ONR/RR-90-1*, 1990a.

[21] Samejima, F. Content-based observation of informative distractors and efficiency of ability estimation. *ONR/RR-90-5*, 1990b.

[22] Samejima, F. and Changas, P. S. How small the number of test items can be for the basis of estimating the operating characteristics of the discrete responses to unknown test items. *ONR/RR-81-3*, 1981.

[23] Samejima, F. and Livingston, P. S. Method of moments as the least squares solution for fitting a polynomial. *ONR/RR-79-2*, 1979.

ONRR9004.TEX
June 25, 1990

Distribution List

Dr. Terry Ackerman
Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Dr. James Algina
1403 Norman Hall
University of Florida
Gainesville, FL 32605

Dr. Erling B. Andersen
Department of Statistics
Studiestraede 6
1455 Copenhagen
DENMARK

Dr. Ronald Armstrong
Rutgers University
Graduate School of Management
Newark, NJ 07102

Dr. Eva L. Baker
UCLA Center for the Study
of Evaluation
145 Moore Hall
University of California
Los Angeles, CA 90024

Dr. Laura L. Barnes
College of Education
University of Toledo
2801 W. Bancroft Street
Toledo, OH 43606

Dr. William M. Bart
University of Minnesota
Dept. of Educ. Psychology
330 Burton Hall
178 Pillsbury Dr., S.E.
Minneapolis, MN 55455

Dr. Isaac Bejar
Mail Stop: 10-R
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

Dr. Menucha Birenbaum
School of Education
Tel Aviv University
Ramat Aviv 69978
ISRAEL

Dr. Arthur S. Blaiwes
Code N712
Naval Training Systems Center
Orlando, FL 32813-7100

Dr. Bruce Bloxom
Defense Manpower Data Center
99 Pacific St.
Suite 155A
Monterey, CA 93943-3231

Cdt. Arnold Bohrer
Sectie Psychologisch Onderzoek
Rekruterings-En Selectiecentrum
Kwartier Koningen Astrid
Bruijnstraat
1120 Brussels, BELGIUM

Dr. Robert Breaux
Code 281
Naval Training Systems Center
Orlando, FL 32826-3224

Dr. Robert Brennan
American College Testing
Programs
P. O. Box 168
Iowa City, IA 52243

Dr. John B. Carroll
409 Elliott Rd., North
Chapel Hill, NC 27514

Dr. John M. Carroll
IBM Watson Research Center
User Interface Institute
P.O. Box 704
Yorktown Heights, NY 10598

Dr. Robert M. Carroll
Chief of Naval Operations
OP-01B2
Washington, DC 20350

Dr. Raymond E. Christal
UES LAMP Science Advisor
AFHRL/MOEL
Brooks AFB, TX 78235

Mr. Hua Hua Chung
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright St.
Champaign, IL 61820

Dr. Norman Cliff
Department of Psychology
Univ. of So. California
Los Angeles, CA 90089-1061

Director, Manpower Program
Center for Naval Analyses
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Director,
Manpower Support and
Readiness Program
Center for Naval Analysis
2000 North Beauregard Street
Alexandria, VA 22311

Dr. Stanley Collyer
Office of Naval Technology
Code 222
800 N. Quincy Street
Arlington, VA 22217-5000

Dr. Hans F. Crombag
Faculty of Law
University of Limburg
P.O. Box 616
Maastricht
The NETHERLANDS 6200 MD

Ms. Carolyn R. Crone
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

Dr. Timothy Davey
American College Testing Program
P.O. Box 168
Iowa City, IA 52243

Dr. C. M. Dayton
Department of Measurement
Statistics & Evaluation
College of Education
University of Maryland
College Park, MD 20742

Dr. Ralph J. DeAyala
Measurement, Statistics,
and Evaluation
Benjamin Bldg., Rm. 4112
University of Maryland
College Park, MD 20742

Dr. Lou DiBello
CERL
University of Illinois
103 South Mathews Avenue
Urbana, IL 61801

Dr. Dattprasad Divgi
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. Hei-Ki Dong
Bell Communications Research
6 Corporate Place
PYA-1K226
Piscataway, NJ 08854

Dr. Fritz Drasgow
University of Illinois
Department of Psychology
603 E. Daniel St.
Champaign, IL 61820

Defense Technical
Information Center
Cameron Station, Bldg 5
Alexandria, VA 22314
(12 Copies)

5/1/90

Dr. Stephen Dunbar
2248 Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. James A. Earles
Air Force Human Resources Lab
Brooks AFB, TX 78235

Dr. Susan Embretson
University of Kansas
Psychology Department
426 Fraser
Lawrence, KS 66045

Dr. George Englehard, Jr.
Division of Educational Studies
Emory University
210 Fishburne Bldg.
Atlanta, GA 30322

ERIC Facility-Acquisitions
2440 Research Blvd, Suite 550
Rockville, MD 20850-3238

Dr. Benjamin A. Fairbank
Operational Technologies Corp.
5825 Callaghan, Suite 225
San Antonio, TX 78228

Dr. Marshall J. Farr, Consultant
Cognitive & Instructional
Sciences
2520 North Vernon Street
Arlington, VA 22207

Dr. P-A. Federico
Code 51
NPRDC
San Diego, CA 92152-6800

Dr. Leonard Feldt
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Richard L. Ferguson
American College Testing
P.O. Box 168
Iowa City, IA 52243

5/1/90

Dr. Gerhard Fischer
Liebiggasse 5/3
A 1010 Vienna
AUSTRIA

Dr. Myron Fischl
U.S. Army Headquarters
DAPE-MRR
The Pentagon
Washington, DC 20310-0300

Prof. Donald Fitzgerald
University of New England
Department of Psychology
Armidale, New South Wales 2351
AUSTRALIA

Mr. Paul Foley
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Alfred R. Fregly
AFOSR/NL, Bldg. 410
Bolling AFB, DC 20332-6448

Dr. Robert D. Gibbons
Illinois State Psychiatric Inst.
Rm 529W
1601 W. Taylor Street
Chicago, IL 60612

Dr. Janice Gifford
University of Massachusetts
School of Education
Amherst, MA 01003

Dr. Drew Gitomer
Educational Testing Service
Princeton, NJ 08541

Dr. Robert Glaser
Learning Research
& Development Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15260

Dr. Sherrie Gott
AFHRL/MOMJ
Brooks AFB, TX 78235-5601

Dr. Bert Green
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

Michael Habon
DORNIER GMBH
P.O. Box 1420
D-7990 Friedrichshafen 1
WEST GERMANY

Prof. Edward Haertel
School of Education
Stanford University
Stanford, CA 94305

Dr. Ronald K. Hambleton
University of Massachusetts
Laboratory of Psychometric
and Evaluative Research
Hills South, Room 152
Amherst, MA 01003

Dr. Delwyn Harnisch
University of Illinois
51 Gerty Drive
Champaign, IL 61820

Dr. Grant Henning
Senior Research Scientist
Division of Measurement
Research and Services
Educational Testing Service
Princeton, NJ 08541

Ms. Rebecca Hetter
Navy Personnel R&D Center
Code 63
San Diego, CA 92152-6800

Dr. Thomas M. Hirsch
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Paul W. Holland
Educational Testing Service,
21-T
Rosedale Road
Princeton, NJ 08541

Dr. Paul Horst
677 G Street, #184
Chula Vista, CA 92010

Ms. Julia S. Hough
Cambridge University Press
40 West 20th Street
New York, NY 10011

Dr. William Howell
Chief Scientist
AFHRL/CA
Brooks AFB, TX 78235-5601

Dr. Lloyd Humphreys
University of Illinois
Department of Psychology
603 East Daniel Street
Champaign, IL 61820

Dr. Steven Hunka
3-104 Educ. N.
University of Alberta
Edmonton, Alberta
CANADA T6G 2G5

Dr. Huynh Huynh
College of Education
Univ. of South Carolina
Columbia, SC 29208

Dr. Robert Jannarone
Elec. and Computer Eng. Dept.
University of South Carolina
Columbia, SC 29208

Dr. Kumar Joag-dev
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright Street
Champaign, IL 61820

Dr. Douglas H. Jones
1280 Woodfern Court
Toms River, NJ 08753

Dr. Brian Junker
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright St.
Champaign, IL 61820

Dr. Michael Kaplan
Office of Basic Research
U.S. Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333-5600

Dr. Milton S. Katz
European Science Coordination Office
U.S. Army Research Institute
Box 65
FPO New York 09510-1500

Prof. John A. Keats
Department of Psychology
University of Newcastle
N.S.W. 2308
AUSTRALIA

Dr. Jwa-keun Kim
Department of Psychology
Middle Tennessee State University
P.O. Box 522
Murfreesboro, TN 37132

Mr. Soon-Hoon Kim
Computer-based Education Research Laboratory
University of Illinois
Urbana, IL 61801

Dr. G. Gage Kingsbury
Portland Public Schools
Research and Evaluation Department
501 North Dixon Street
P. O. Box 3107
Portland, OR 97209-3107

Dr. William Koch
Box 7246, Meas. and Eval. Ctr.
University of Texas-Austin
Austin, TX 78703

Dr. Richard J. Koubek
Department of Biomedical & Human Factors
139 Engineering & Math Bldg.
Wright State University
Dayton, OH 45435

Dr. Leonard Kroeker
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Dr. Jerry Lehnus
Defense Manpower Data Center
Suite 400
1600 Wilson Blvd
Rosslyn, VA 22209

Dr. Thomas Leonard
University of Wisconsin
Department of Statistics
1210 West Dayton Street
Madison, WI 53705

Dr. Michael Levine
Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Dr. Charles Lewis
Educational Testing Service
Princeton, NJ 08541-0001

Mr. Rodney Lim
University of Illinois
Department of Psychology
603 E. Daniel St.
Champaign, IL 61820

Dr. Robert L. Linn
Campus Box 249
University of Colorado
Boulder, CO 80309-0249

Dr. Robert Lockman
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. Frederic M. Lord
Educational Testing Service
Princeton, NJ 08541

Dr. Richard Luecht
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. George B. Macready
Department of Measurement Statistics & Evaluation
College of Education
University of Maryland
College Park, MD 20742

Dr. Gary Marco
Stop 31-E
Educational Testing Service
Princeton, NJ 08451

Dr. Clessen J. Martin
Office of Chief of Naval Operations (OP 13 F)
Navy Annex, Room 2832
Washington, DC 20350

Dr. James R. McBride
The Psychological Corporation
1250 Sixth Avenue
San Diego, CA 92101

Dr. Clarence C. McCormick
HQ, USMEPCOM/MEPCT
2500 Green Bay Road
North Chicago, IL 60064

Mr. Christopher McCusker
University of Illinois
Department of Psychology
603 E. Daniel St.
Champaign, IL 61820

Dr. Robert McKinley
Educational Testing Service
Princeton, NJ 08541

Mr. Alan Mead
c/o Dr. Michael Levine
Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Dr. Timothy Miller
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Robert Mislevy
Educational Testing Service
Princeton, NJ 08541

Dr. William Montague
NPRDC Code 13
San Diego, CA 92152-6800

Ms. Kathleen Moreno
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Headquarters Marine Corps
Code MPI-20
Washington, DC 20380

Dr. Ratna Nandakumar
Educational Studies
Willard Hall, Room 213E
University of Delaware
Newark, DE 19716

Library, NPRDC
Code P201L
San Diego, CA 92152-6800

Librarian
Naval Center for Applied Research in Artificial Intelligence
Naval Research Laboratory
Code 5510
Washington, DC 20375-5000

Dr. Harold F. O'Neil, Jr.
School of Education - WPH 801
Department of Educational
Psychology & Technology
University of Southern
California
Los Angeles, CA 90089-0031

Dr. James B. Olsen
WICAT Systems
1875 South State Street
Orem, UT 84058

Office of Naval Research,
Code 1142CS
800 N. Quincy Street
Arlington, VA 22217-5000
(6 Copies)

Dr. Judith Orasanu
Basic Research Office
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Jesse Orlansky
Institute for Defense Analyses
1801 N. Beauregard St.
Alexandria, VA 22311

Dr. Peter J. Pashley
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

Wayne M. Patience
American Council on Education
GED Testing Service, Suite 20
One Dupont Circle, NW
Washington, DC 20036

Dr. James Paulson
Department of Psychology
Portland State University
P.O. Box 751
Portland, OR 97207

Dept. of Administrative Sciences
Code 54
Naval Postgraduate School
Monterey, CA 93943-5026

Dr. Mark D. Reckase
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Malcolm Ree
AFHRL/MOA
Brooks AFB, TX 78235

Mr. Steve Reiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455-0344

Dr. Carl Ross
CNET-PDCD
Building 90
Great Lakes NTC, IL 60088

Dr. J. Ryan
Department of Education
University of South Carolina
Columbia, SC 29208

Dr. Fumiko Samejima
Department of Psychology
University of Tennessee
310B Austin Peay Bldg.
Knoxville, TN 37916-0900

Mr. Drew Sands
NPRDC Code 62
San Diego, CA 92152-6800

Lowell Schoer
Psychological & Quantitative
Foundations
College of Education
University of Iowa
Iowa City, IA 52242

Dr. Mary Schratz
905 Orchid Way
Carlsbad, CA 92009

Dr. Dan Segall
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Robin Shealy
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright St.
Champaign, IL 61820

Dr. Kazuo Shigemasu
7-9-24 Kugenuma-Kaigan
Fujisawa 251
JAPAN

Dr. Randall Shumaker
Naval Research Laboratory
Code 5510
4555 Overlook Avenue, S.W.
Washington, DC 20375-5000

Dr. Richard E. Snow
School of Education
Stanford University
Stanford, CA 94305

Dr. Richard C. Sorensen
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Judy Spray
ACT
P.O. Box 168
Iowa City, IA 52243

Dr. Martha Stocking
Educational Testing Service
Princeton, NJ 08541

Dr. Peter Stoloff
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. William Stout
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright St.
Champaign, IL 61820

Dr. Hariharan Swaminathan
Laboratory of Psychometric and
Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003

Mr. Brad Sympson
Navy Personnel R&D Center
Code-62
San Diego, CA 92152-6800

Dr. John Tangney
AFOSR/NL, Bldg. 410
Bolling AFB, DC 20332-6448

Dr. Kikumi Tatsuoka
Educational Testing Service
Mail Stop 03-T
Princeton, NJ 08541

Dr. Maurice Tatsuoka
220 Education Bldg
1310 S. Sixth St.
Champaign, IL 61820

Dr. David Thissen
Department of Psychology
University of Kansas
Lawrence, KS 66044

Mr. Thomas J. Thomas
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

Mr. Gary Thomasson
University of Illinois
Educational Psychology
Champaign, IL 61820

Dr. Robert Tsutakawa
University of Missouri
Department of Statistics
222 Math. Sciences Bldg.
Columbia, MO 65211

Dr. Ledyard Tucker
University of Illinois
Department of Psychology
603 E. Daniel Street
Champaign, IL 61820

Dr. David Vale
Assessment Systems Corp.
2233 University Avenue
Suite 440
St. Paul, MN 55114

Dr. Frank L. Vicino
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Howard Wainer
Educational Testing Service
Princeton, NJ 08541

Dr. Michael T. Waller
University of Wisconsin-Milwaukee
Educational Psychology Department
Box 413
Milwaukee, WI 53201

Dr. Ning-Mei Wang
Educational Testing Service
Mail Stop 03-T
Princeton, NJ 08541

Dr. Thomas A. Warm
FAA Academy AAC934D
P.O. Box 25082
Oklahoma City, OK 73125

Dr. Brian Waters
HumRRO
1100 S. Washington
Alexandria, VA 22314

Dr. David J. Weiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455-0344

Dr. Ronald A. Weitzman
Box 146
Carmel, CA 93921

5/1/90

Major John Welsh
AFHRL/MOAN
Brooks AFB, TX 78223

Dr. Douglas Wetzel
Code 51
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Rand R. Wilcox
University of Southern California
Department of Psychology
Los Angeles, CA 90089-1061

German Military Representative
ATTN: Wolfgang Wildgrube
Streitkraefteamt
D-5300 Bonn 2
4000 Brandywine Street, NW
Washington, DC 20016

Dr. Bruce Williams
Department of Educational Psychology
University of Illinois
Urbana, IL 61801

Dr. Hilda Wing
Federal Aviation Administration
800 Independence Ave, SW
Washington, DC 20591

Mr. John H. Wolfe
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. George Wong
Biostatistics Laboratory
Memorial Sloan-Kettering Cancer Center
1275 York Avenue
New York, NY 10021

Dr. Wallace Wulfeck, III
Navy Personnel R&D Center
Code 51
San Diego, CA 92152-6800

Dr. Kentaro Yamamoto
02-T
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

Dr. Wendy Yen
CTB/McGraw Hill
Del Monte Research Park
Monterey, CA 93940

Dr. Joseph L. Young
National Science Foundation
Room 320
1800 G Street, N.W.
Washington, DC 20550

Mr. Anthony R. Zara
National Council of State Boards of Nursing, Inc.
625 North Michigan Avenue
Suite 1544
Chicago, IL 60611